

Lec 10

MA(1) Invertibility

$$Y_t = W_t + \theta v_{t-1}$$

$$v_t = Y_t - \theta v_{t-1}$$

$$v_t = Y_t - \theta (Y_{t-1} - \theta v_{t-2})$$

$$= Y_t - \theta Y_{t-1} + \theta^2 v_{t-2}$$

$$= Y_t + \sum_{i=1}^{\infty} (-\theta)^i Y_{t-i}$$

$$= Y_t + \sum_{i=1}^p (-\theta)^i Y_{t-i} + (-\theta)^{p+1} Y_{t-(p+1)}$$

$$Y_t = \mu_t + v_t$$

$$\mu_t = \mu_{t-1} + v_t$$

$$\mu_0 = 0$$

$$\mu_1 = v_1$$

$$\mu_2 = v_1 + v_2$$

⋮

$$\mu_t = \sum_{i=1}^t v_i$$

$$Y_0 = 0$$

$$Y_1 = v_1 + w_1$$

$$Y_2 = v_1 + v_2 + w_2$$

⋮

$$Y_t = \sum_{i=1}^t v_i + w_t$$

$$E(Y_t) = 0$$

$$\text{Var}(Y_t) = t \sigma_v^2 + \sigma_w^2$$

$$\begin{aligned} \Delta Y_t &= Y_t - Y_{t-1} = (\mu_t + w_t) - (\mu_{t-1} + w_{t-1}) \\ &= \sum_{i=1}^t v_i + w_t - \sum_{i=1}^{t-1} v_i - w_{t-1} \\ &= v_t + w_t - w_{t-1} \end{aligned}$$

$$E(\Delta Y_t) = 0$$

$$\text{Var}(\Delta Y_t) = \sigma_v^2 + 2\sigma_w^2$$

$$\text{Cov}(\Delta Y_t, \Delta Y_{t-h}) = \begin{cases} \sigma_v^2 + 2\sigma_w^2 & \text{if } h=0 \\ \sigma_v^2 & \text{if } h=1 \\ 0 & \text{if } h > 1 \end{cases}$$

$$E\left((v_t + w_t + w_{t-1}) (v_{t-1} + w_{t-1} + w_{t-2}) \right)$$

$$Y_t = M_t + W_t \quad M_t = M_{t-1} + V_t \quad V_t = V_{t-1} + e_t$$

$$V_0 = 0$$

$$M_0 = 0$$

$$V_1 = e_1$$

$$M_1 = e_1$$

$$V_2 = e_1 + e_2$$

$$M_2 = e_1 + V_2 = 2e_1 + e_2$$

$$V_3 = e_1 + e_2 + e_3$$

$$M_3 = 2e_1 + e_2 + (e_1 + e_2 + e_3)$$

⋮

$$= 3e_1 + 2e_2 + e_3$$

$$V_t = \sum_{i=1}^t e_i$$

⋮

$$M_t = \sum_{i=1}^t (t+1-i) e_i$$

$$\Delta Y_t = (M_t + V_t) - (M_{t-1} + V_{t-1})$$

$$= (M_t - M_{t-1}) + (V_t - V_{t-1})$$

$$= (M_{t-1} + V_t - M_{t-1}) + (V_t - V_{t-1})$$

$$= V_t + (V_t - V_{t-1})$$

$$= \sum_{i=1}^t e_i + (V_t - V_{t-1})$$

$$\Delta^2 y_t = [V_t + (w_t - w_{t-1})] - [V_{t-1} + (w_{t-1} - w_{t-2})]$$

$$= V_{t-1} + e_t - V_{t-1} + w_t - 2w_{t-1} + w_{t-2}$$

$$= e_t + w_t - 2w_{t-1} + w_{t-2}$$